Quantum Determination of Elliptical Periphery and the Detection of Systemic Error in the Maclaurin Derivative Series

Lawrence Dawson

Submitted to the Virtual University in completion of the Masters Degree in Mathematics

The Strict Euclidean Dilemma for the Ellipse

Calculation of Circumference is inexact: Solved by Incorporating the Quantum Dimension

NOTE: A level of mathematical knowledge is required of the reader. It is assumed the reader will know how an ellipse is constructed, the relationship between focal point and major and minor axis, and what the statistical measure "eccentricity" is.

Quantum mathematics provides an exact measure of the circumference or periphery of an ellipse. It does so by an analysis of the variance in curvature. Thus, quantum geometry solves a problem which has eluded strict Euclidean mathematics for three hundred years. No Eulcidean-derived formula has provide an exact measure of the periphery of the ellipse and this includes the best of the Euclidean-based formulas, that of Colin Maclaurin.

The quantum-derived formula is the following:

$\chi =$ circumference of ellipse ; $r_1 =$ minor axis ; $r_2 =$ major axis

$$\chi = 2\sqrt{3\,r_1^2 + r_2^2}\left(\frac{2\,r_1}{\sqrt{r_1^2 + 3\,r_2^2}}\left(\frac{\pi - 3}{3}\right) + 1\right) + 2\left(r_1\left(\frac{\pi - 3}{3}\right) + r_2\right)$$

For proof of equality see Appendix; "Determining the Circumference of an Ellipse by Strict Quantum Construction"

This exact formula is possible because quantum dimensional analysis of an ellipse can identify variances in curvature as the circle becomes an ellipse.

Quantum circumference is composed of a summation of straight-line secants. The variance between the straight quantum lines and the curved Euclidean lines of the circle is calculable. Further, as the circle is modified to become an ellipse, the quantum secants and the curvature variance change in a mathematically regular way. The above formula is the summation of these changes in curvature variance.

The factor "(π-3)/3" in the formula is the curvature-variance constant for an ellipse treated as a quantum construction. The formula interfaces "quantum π" (3) with "Euclidean π" (3.1416). Quantum geometric construction gives a different "π" value because quantum circumference of a circle is defined as the summation of straight-line secants, not the curved line of the Euclidean circle.[1]

The curvature -variance constant is utter nonsense for an ellipse of strict Euclidean construction; that is, of an ellipse which recognize only a single π value.

This is not to say that the standard Euclidean construction is in error. Rather, Euclidean construction is an incomplete mathematical description of the ellipse. Euclidean geometry knows nothing about the quantum secants which are rigorously defined on the graph of the ellipse; their mathematical values being strictly formulated by the minor and major axis. Nor does Euclidean math know that curvature constant is also strictly formulated by these mathematics.

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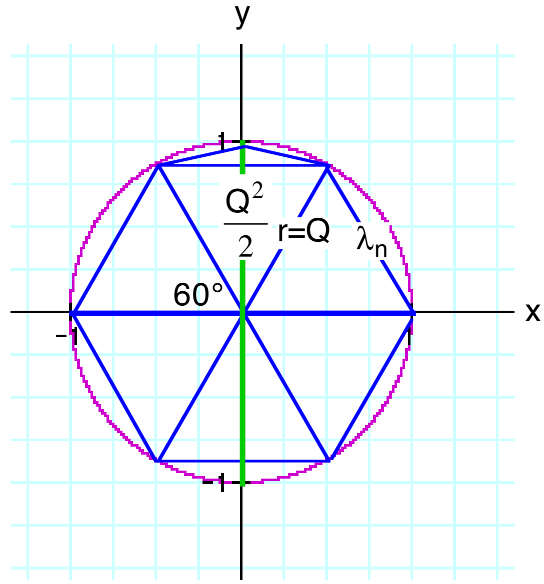
[1] Quantum circumference is composed of six secants all of which are equal to the radius. Therefore, quantum circumference is equal to "6r." Since π is the ratio of circumference to diameter, quantum π is equal to "3."

Curvature is the change in slope ($d(y)/d(x)$) for every point along the circumference, or, more properly "for every moment of angular motion." Variance in curvature is the difference between the change in slope for an ellipse and the change of slope for a circle of equal angle and radius.

Strict Euclidean geometry cannot identify the variance between the change of slope for a circle and the change of slope for the ellipse. Why this is true is a major portion of substance of this paper.

To recognize how quantum geometry fortifies strict Euclidean geometry requires that we begin with a circle. In the figure below, the quantum imposes its own form upon the circle. The hexagram appears to be a second, auxiliary figure subscribed within the circle. In point of fact, it is actually an alternative definition of the circle. There is, however, a variance between Euclidean defined-circumference and the quantum-defined circumference. Quantum circumference equals the sum of the secants and therefore has a "π" value equal to "3."

The Quantum Intersected Circle



Euclidean Circumference = $\chi_E = 2\,\pi\,r$; secant = λ_n ; Q = quantum

Quantum Circumference = $\chi_Q = \sum_{n=1}^{6} \lambda_n$; r = radius = 1 = λ_n

$\pi = (\text{circumference}) / 2(\text{radius}) = 3.1416$; $\pi_Q = 6(\text{radius}) / 2(\text{radius}) = 3$

Quantum Circumference = $6\,r$; Euclidean Circum. = $2\,\pi\,r = 6.2831853072\,r$

Statistical curvature constant (*per secant) =* $(\lambda_n)(\pi - 3) / 3 = 0.0471975512(\lambda_n)$

Amount which must be added to secant to equal the curved line length

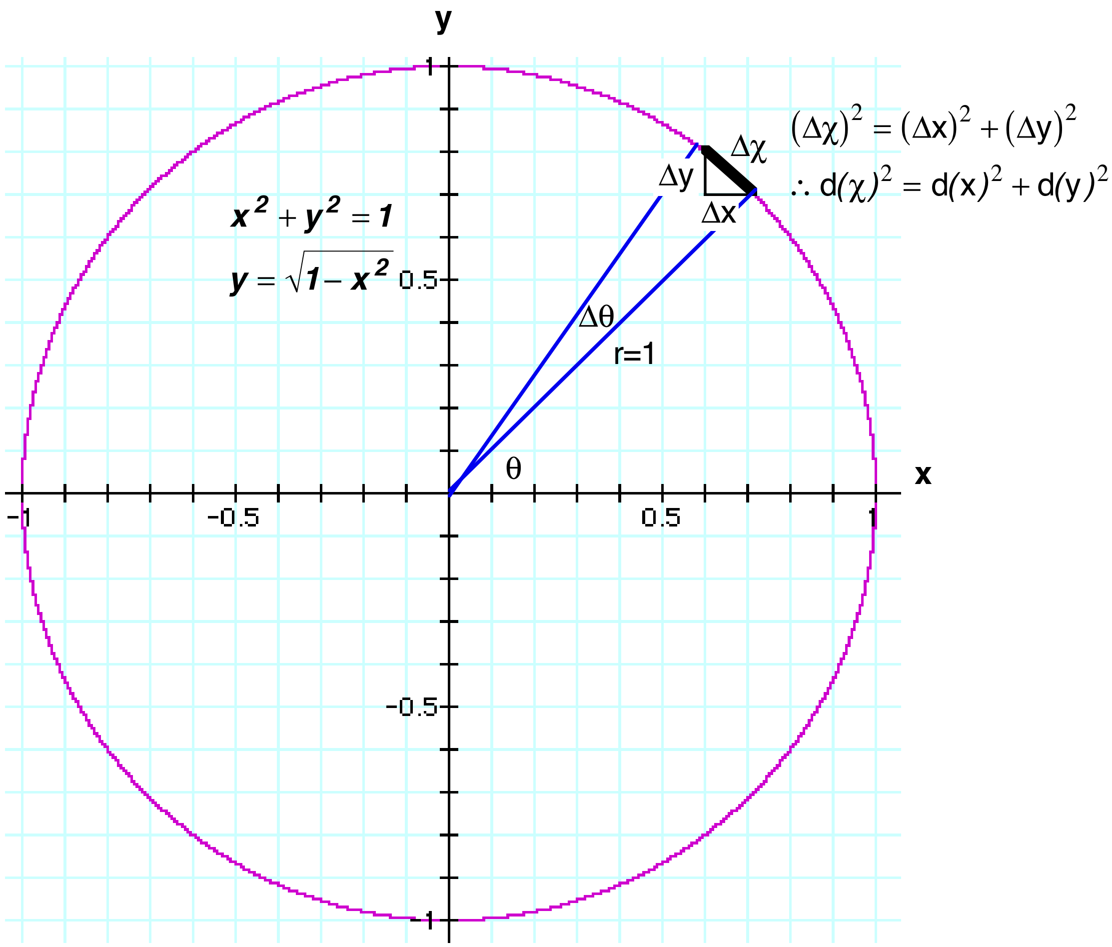
Curvature Constant

Statistical curvature constant *times* the length of the secant is amount which must be added to the secant to equal the length of the curved line subscribing that secant. With the strict Euclidean circle, a statistical curvature constant is always problematic. The quantum curvature constant can be modified to determine the distance of the curved line subscribing the secant. This is not possible with a strict Euclidean defined circle.

The Strict Euclidean Treatment Cannot Identify Curvature Variance

The curved line of the Euclidean circumference of a circle is achieved by differentiating both the “x” axis and the “y” axis to “0.”

With this conventional, “non quantum” treatment of circumference, all curvature variance information drops out. That strict Euclidean treatment is summarized below:



$x = r\,cos(\theta)$; $y = r\,sin(\theta)$; r = radius ; χ = length of curved line

$$d(x) = -r\,sin(\theta)\,d(\theta) \;;\; d(y) = r\,cos(\theta)\,d(\theta)$$

$$d(\chi)^2 = d(x)^2 + d(y)^2 = r^2\,sin(\theta)^2\,d(\theta)^2 + r^2\,cos(\theta)^2\,d(\theta)^2$$

$$d(\chi)^2 = r^2\left(sin(\theta)^2 + cos(\theta)^2\right)d(\theta)^2 = r^2(1)\,d(\theta)^2 \;;\; \left\{sin(\theta)^2 + cos(\theta)^2 = 1\right\}$$

$$d(\chi) = \sqrt{r^2(1)\,d(\theta)^2} = r\,d(\theta)d(\chi) = \sqrt{r^2(1)\,d(\theta)^2} = r\,d(\theta)$$

$$\chi = \pi\,2\,r = \int_{\theta=0}^{360^\circ} r\,d(\theta)$$

The change in angle ($\Delta\theta$) creates a change in length of the curved line ($\Delta\chi$). For the purposes of calculus, "$\Delta\chi$" can be considered as the secant of the curved line. This is possible because length of secant approaches length of curvature as the differential approaches "0" . Now, as the length of the secant, "$\Delta\chi$" approaches "0" any variance between the secant and the curved line also approaches "0."

$$\text{As } \Delta\chi \xrightarrow{\text{appr.}} (0 = d(\chi)),$$

$$\text{Statistical curvature variance } (\textit{per secant}) \xrightarrow{\text{appr.}} 0 \;;$$

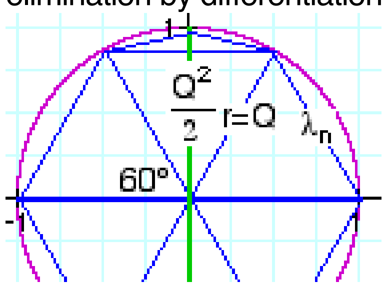
That is, all information about curvature variance is lost completely. All quantum dimensional values have been eliminated.

Differential Equation Identify the Eliminated Quantum Dimension

The calculus treatment of the circle's circumference, as given above, identifies the very quantum dimension which it has eliminated. The radius must become a quantum in order to construct circumference.

The circumference as constructed above would more properly be identified as an "orbit." The radius must be composed of two points separated by a fixed distance. If the radius were a solid line composed of a continuum of points, the rotation of that solid line would construct area. Two points separated by a distance is the definition of a quantum. The line so defined is rotated around one of the end points and the other point constructs a curved linear path of distance "χ."

The radius cannot be a quantum unless the differentiation of the angle "θ" has unconsciously addressed the quantum dimension. The differentiation of "θ" to "0" ($d(\theta) = 0$) must be the unrecognized elimination by differentiation of the quantum 60° angle.



I will digress for a moment into a basic principle of quantum geometry. The unit of area called the "quantum squared" ($Q^2/2$ above) cannot be a two dimensional quantum. Since there is only one quantum dimension, the quantum squared is defined by the one quantum axis (the radius) and a second Euclidean axis (the secant). The quantum radius is integrated across the Euclidean secant to construct the quantum squared. The linear quantum cannot be reacquired by taking the square root of the unit of area since the resultant factor cannot be the quantum. Only "anti-integration" —or the derivative— can produce the quantum:

$$D(Q^2/2) = Q$$

The "anti-integration" (derivative) of the quantum squared eliminates the 60° angle remaindering the linear quantum radius. The differentiation of the angle θ to "0" is the equivalent of this anti-integration.

It is easily proven that the radius which constructs circumference must be the quantum. The circumference of a circle is the derivative of the area subscribed by the circle.

$$Area = \pi r^2 \text{ ; circumference } = 2\pi r = \chi$$

$$D(\pi r^2) = \frac{d(\pi r^2)}{d(r)} = 2\pi r$$;***Circumference = the derivative of area.***

$$d(\pi r^2) = 2\pi r\, d(r)$$

$$\pi r^2 = \int_0^r 2\pi r\, d(r) = \int_0^r \chi\, d(r)$$;***Area = the integration of circumference by radius***

Area is constructed by integrating circumference by radius and thus composing radius as a continuum of points. This integration converts the radius to an Euclidean line. The integration produces a solid-line of strict Euclidean definition.

The quantum therefore is the unrecognized substructure of the Euclidean circle. The order of its construction is the following:

1)Starting with $Q^2/2$, the quantum angle is anti-integrated to “0” (resulting in $d(\theta)$). This anti-integration of the angle produces the quantum as a radius and is the only way the radius as quantum can be produced.

2)This quantum radius is then integrated by “ $d(\theta)$” from 0° to 360° producing the circumference of the circle.

3) The circumference of the circle, in turn, is integrated by the radius from “r=0 to r=r.” This converts the quantum radius to an Euclidean radius and the circumference as subscribing Euclidean area.

By construction, the strict Euclidean circle has lost a degree of definition, that is recognition of its quantum component. Lost is all information on the curvature variance between the quantum secants and the curved circumference. This information can be required by renewing the quantum definition.

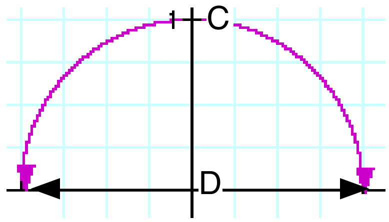
Loss of Curvature Variance for the Ellipse

The only curvature-to-linear information retained by strict Euclidean circle construction is the value of pi (π). Π is the ratio of the curved circumference to the linear diameter of a circle. Π is 2 *times* the curved length subscribing the diameter

$$\frac{\pi}{2} = \frac{C}{D} = \text{ unit of curved length per unit of straight length}$$

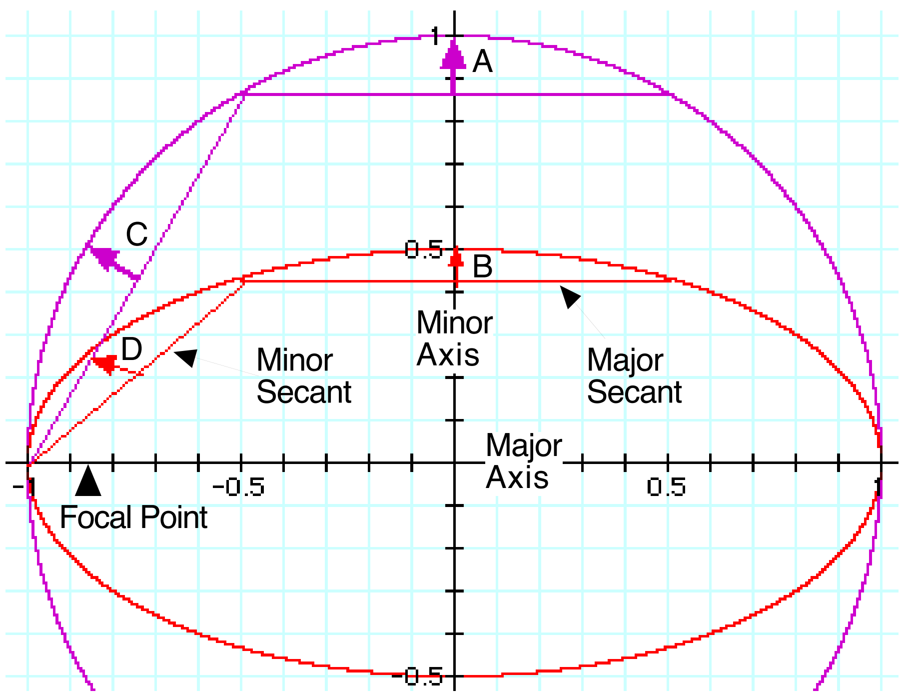
$$= 1.5708 \text{ curved units } per \text{ straight unit}$$

A ratio of the curved line "C" subscribing the straight diameter "D."



While π is an adequate curvature variance statistic for the circle, it is completely inadequate for the ellipse. Quantum construction shows why this is true.

Ellipse: focal point= .866
major axis= 1
minor axis= .5



The above illustration shows how the quantum secants of a circle are changed by an ellipse with an eccentricity[2] defined by the circle. Curvature variance is establish by the distance to the "high point" of the curved line which subscribes the quantum secant. High-point distance is identified by the radius which bisects the secant.

The mathematical details covering these relationships are somewhat complex and

[2] Eccentricity=(focal point) *divided by* (circle's radius=ellipse's major axis).

examined in the appendix. Without going into these details, however, the graphic can illustrate what is happening to curvature variance for the ellipse *vis a vis* its originating circle.

The lengths labeled “A,” “B,” “C” and “D” are the distances to the curvature high points along the radius bisecting the secant. Those lengths determine curvature variance. The length of elliptical secant “B” is shorter than the equivalent circular length of “A.” That is, the curvature variance for the ellipse’s major secant[3] is less than the curvature variance for the equivalent circle secant.

Although more complex this is also true for the minor secant[4] . The ellipse’s minor secant curvature variance “D” is less than the equivalent circle secant curvature variance of “C”.

The quantum defined ellipse is a systematic compression of the quantum defined circle, leaving a mathematically regularized set of quantum elliptical secants and curvature variances for those secants.

The elliptical curvature variances are not the same as the circular curvature variances. All elliptical curvature variances have been compressed along the minor axis just as the ellipse itself is compressed along the minor axis.

All elliptical curvatures vary by fractions as determined by the minor axis. These fractions modify the statistical curvature constant. (see Appendix). This constant is based upon both Euclidean and quantum “π.”

The assumption that Euclidean “π” alone can determine a valid curvature constant for the ellipse is the major source of error for Euclidean based estimations of elliptical circumference. I will let the best of these estimations, that of Colin Maclaurin, show that this is true.

[3] The major secant is the quantum secant of fixed length equal to the major axis radius.

[4] The minor secant is the quantum secant of variable length as a function of the minor radius. The formula for the minor secant is:

$$\sqrt{3\,(\text{minor radius})^2 + (\text{major radius})^2}$$

Systematic Error in the Maclaurin Series Estimate of Elliptical Circumference

With no direct Euclidean geometric formula for elliptical circumference, that circumference is estimated by the Maclaurin Series. Colin Maclaurin[5] offered an estimation of the integral of the *unknown function* for elliptical circumference based upon the *known factor* of the ellipse's "eccentricity."

The Maclaurin equation is the "gold standard" of strict Euclidean treatment of elliptical circumference. Although modernists have offered many "modifications," all are empirical equations with nothing of Maclaurin's rigorous calculus. We will stick with Maclaurin and assume his derivative series , in fact, correctly estimates an *unknown* integral for elliptical circumference.

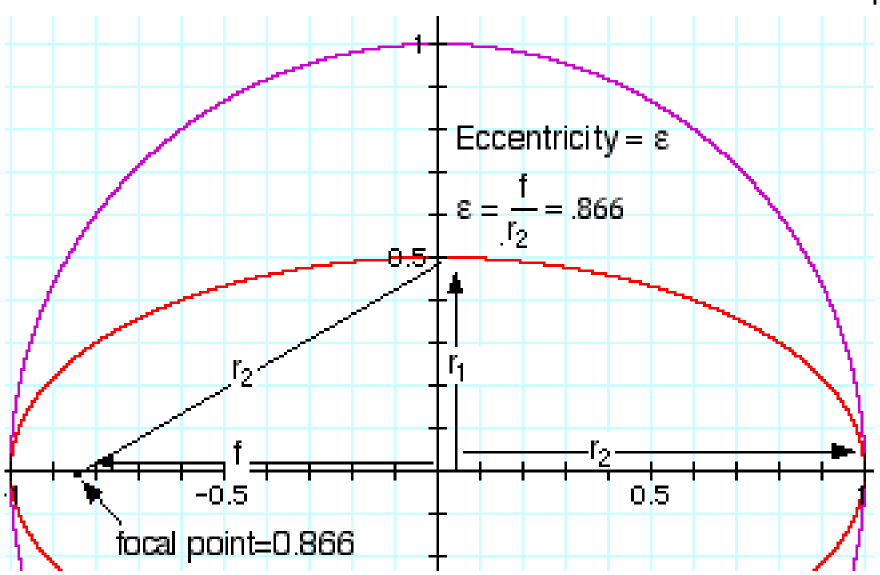
Maclaurin's equation for the ellipse is a derivative-series estimation of the integral of an *unknown function* for the formula for the circumference of a circle which converts it to the perimeter of an ellipse. The *known factor* which Maclaurin uses for his *unknown function* is elliptical eccentricity.

The Maclaurin formula is based upon the Taylor Theorem[6] . The Taylor Theorem gives an approximation of an *unknown function* by using the derivatives of a *known factor* for the *unknown function*. The Taylor expansion of derivatives uses factorials.(Factorials are number multiplied by all lower orders of themselves (e.g. 3! = 3•2•1). The factorials sum the derivatives to a specialized mean value for the integral. The Maclaurin equation is a Taylor estimation of an *unknown function* for elliptical circumference.

The *known factor* which Maclaurin uses for his Taylor estimation of circumference is elliptical eccentricity.

The eccentricity is the ellipse's focal point as a percentage of the major axis radius.

$$f = \text{focal point} \; ; \quad f^2 = r_2^2 - r_1^2 \; ; \; f = \sqrt{r_2^2 - r_1^2} \; ; \; \varepsilon = \frac{f}{r_2}$$



[5] 1698-1746. Professor Mathematics, University of Edinburg. Protégé and student of Isaac Newton. Author of Maclaurin Series estimations of unknown functions for partials of a whole.
[6] Brook Taylor, 1712.

Eccentricity is a measure of an ellipse's variance from its originating circle (see above). For the circle: $r_1 = r_2$ and $r_2^2 - r_1^2 = 0$; therefore f = 0. The eccentricity (f/ r_2) also equals "0." There is "0" variance between the circle and the ellipse.

When the circle is compressed completely to the major "r_2" axis, "$r_1 = 0$" and eccentricity =1(f=r_2). Eccentricity, then, is a measure of a circle's elliptical compression and ranges from "0" compression (the circle) to "1" (complete compression).

Maclaurin is arguing that there is an *unknown function* of eccentricity which relates the formula for the circumference of a circle to the periphery of the ellipse. While the function is unknown, its general form or outline and some of its factors can be determined. The formula for the circle's circumference and a curvature constant based upon Euclidean π only are two such determined factors Maclaurin uses. Elliptical periphery is presumed to result from the *unknown function* of eccentricity operating upon these known factors and forms.

Maclaurin doesn't care what this unknown function of eccentricity actually is. He has his Taylor Theorem derivative series, the summation of which estimates the integral of the derivative of the unknown function, whatever that function may be.

The Taylor Theorem holds that factorial modifications of derivatives can be used to estimate a mean value of the integral for the *unknown function.* This is accomplished by summation of the derivative series for the *known factor*, as modified by the factorials. . The mean value thus estimated is the integral of the *unknown function.*

The integration of the derivative of any function equals the function. Maclaurin's derivative-series estimation of the integral thus equals the *unknown function*.

However, Maclaurin had to make serious modifications to the Taylor factorials because his *known factor*— eccentricity — is a partial or fraction of a whole. The derivative series of a fraction of a whole presented special problems.

Some function of eccentricity which modifies the circumferences of a circle and Maclaurin's curvature constant is presumed to render elliptical circumference. Eccentricity is a partial of a whole in that it is a fraction laying somewhere between "0" and "1." It can be defined as "1/ x" where "x" is any number between 1 and infinity.

A derivative series is built upon the fact that a derivative of a function is also a function which has a derivative which is a function which also has a derivative......and so forth. These are called the "1st, 2nd, 3rd...so forth" derivatives. The mathematical symbol for this is the following:

$$D^n(f(x)) \text{ where "n" is the "nth" derivative of the function of x.}$$

The problem with the derivative series for eccentricity as a fraction of a whole is that the derivatives alternate between the derivative for the fraction (1/ x) and the derivative for the negation of the fraction (1-1/ x). This makes Taylor Theorem summation of the derivative series impossible:

$$D^n(1/x) = \pm\frac{n!}{x^{n+1}}$$

$$D^1(1/x) = -\left(\frac{1}{x}\right)^2 \;;\; D^2(1/x) = 2\left(\frac{1}{x}\right)^3 \;;\; D^3(1/x) = -6\left(\frac{1}{x}\right)^4 \;;\; D^4(1/x) = 24\left(\frac{1}{x}\right)^5 \;;\; \text{....etc.}$$

Notice that the derivative series alternates between negative values and positive values. All even number derivative exponential (i.e. 2,4,6,8 etc.) are negative. All odd number derivative exponentials (i.e. 3,5,7, etc.) are positive. In a straight forward summation of the derivative series, each successive derivative would be subtracted from the value, rendering the estimation incorrect.

Maclaurin solved the problem by restricting the derivatives to the negative values (even-numbered exponential). Maclaurin's derivative series for the estimation of elliptical circumference is the following:

$$\varepsilon = \text{eccentricity} = \frac{f}{r_2}$$

$$\chi_e = 2\pi r_2 \left(1 - \sum_1^n \frac{1}{2n-1} \left(\frac{(2n)!}{\left(2^n n!\right)^2} \right)^2 \varepsilon^{2n} \right) \quad ; \text{ n = is a derivative value of the partial "}\varepsilon\text{"}$$

which expands to:

$$\chi_e = 2\pi r_2 \left(1 - \left[\left(\frac{1}{2}\right)^2 \varepsilon^2 + \left(\frac{1\bullet 3}{2\bullet 4}\right)^2 \frac{\varepsilon^4}{3} + \left(\frac{1\bullet 3\bullet 5}{2\bullet 4\bullet 6}\right)^2 \frac{\varepsilon^6}{5} + \left(\frac{1\bullet 3\bullet 5\bullet 7}{2\bullet 4\bullet 6\bullet 8}\right)^2 \frac{\varepsilon^8}{7} \ldots\ldots\ldots\ldots \right] \right)$$

Proof: expansion of Maclaurin Factorial for " n = 2."

$$\frac{(1**3*)(1**3*)}{(**1)(**1)\left(2^2*2*1\right)^2} \frac{\varepsilon^4}{3} = \frac{1}{2n-1} \left(\frac{(2n)!}{\left(2^n n!\right)^2} \right)^2 \varepsilon^{2n}$$

Notice that Maclaurin's exponentials of eccentricity are restricted to even numbered exponentials (ε^{2n}). He is restricting the derivative series to the negative derivatives.

The Maclaurin derivative series estimates the change which an unknown function of elliptical eccentricity makes to the circumference of a circle and to a curvature constant based upon π alone. This curvature constant has guided Maclaurin's choice for the factorials which modify the derivative series. The unknown function of elliptical eccentricity is presumed to modify circular circumference and the π-based curvature constant.

Therefore, π and π alone is presumed to be the curvature constant for all ellipses and this introduces systematic error into the Maclaurin equation. Quantum geometry has proven that π must be modified by "quantum π" to accurately determine the curvature variance for an ellipse. The Maclaurin formula, with a "π alone" curvature constant cannot accurately account for this change in curvature variance.

Maclaurin's formula is "1 *minus* the factorial modified derivative series *times* circular circumference." As eccentricity gets larger their is a greater variance of the ellipse from the originating circle. As eccentricity increases, the value of the derivative series also increased, making the remainder of its subtraction from "1" a smaller number. This smaller number is multiplied by circular circumference giving a smaller elliptical circumference.

The systematic error which the formula makes in assuming a π-based curvature constant is shown in the table below. The comparison between the exact, quantum-determined periphery and Maclaurin's estimations shows that Maclaurin's error varies around the median of possible minor axis values. Maclaurin's estimates are higher than the actual periphery.

Eccentricity: $r_2 = 1$	Quantum Curvature Variance ***var⁄circ.***	**Quantum Determined Periphery**	difference	**Maclaurin Estimated Periphery.** ***% error over quantum*** *number of required diff.*
0	0.047198 ***1***	2π	0	2π ***+0%*** *n=0*
0.25	0.045938 ***0.973***	6.1763929187	0.0074361058	6.1838290245 ***+.120%*** *n=6*
0.5	0.041736 ***0.884***	5.8395073613	0.0303414761	5.8698488374 ***+.519%*** *n=13*
0.8660254038	0.025069 ***0.531***	4.7622159305	0.082010983	4.8442269135 ***+1.722*** *n=26*
0.95	0.015807 ***0.335***	4.3413177743	0.0697074134	4.4110251877 ***+1.61%*** *n=35*
1	0 ***0***	4	0.019850751	4.019850751 ***+.50%*** *n=50*

Systemic Error in the Maclaurin Formula

The above data table can be used to demonstrate that the Maclaurin Series elliptical estimate suffers systemic error because it employs a meaningless curvature constant.

The table shows elliptical circumference calculations for six different eccentricities and how the quantum calculation differs in result from the Maclaurin estimate. For all eccentricities greater than 0, Maclaurin's estimations of elliptical circumference is higher than the exact value as determined by the quantum.

For all eccentricities greater than "0," the Maclaurin formula gives slightly larger circumference values than the quantum formula.

Quantum curvature variance for each ellipse as a percentage of curvature variance for the circle is also given in the table. For example, an ellipse with an eccentricity of ".25" has a quantum curvature variance which is 97.3% of circular curvature variance. This is a slightly compressed ellipse, relative to the circle. The minor axis is only .866 of major axis.

"Curvature variance" is determined by the quantum curvature constant. This constant is built upon the difference between π and quantum π[7]. Modification of the constant determines the amount the curved line subscribing the quantum secant varies from that secant. As the the ellipse becomes more compressed from the circle its eccentricity goes up and the amount the curved line varies from the quantum secant, its "curvature variance," goes down. There is less average curvature relative to the quantum secants for more compressed ellipses.

As quantum curvature variance goes down, Maclaurin's "error" with respect to actual circumference goes up. At "ε=.25," curvature variance is 97.3% of circular and Maclaurin's estimate of circumference is .12% greater than the quantum determination. At "ε=.5,"

[7] π *minus* quantum π *divided by* quantum π.

curvature variance is 88.4% of circular and Maclaurin circumference is .519% greater than quantum.

This is true only up to a point. Maclaurin circumferential “error” varies around the median minor axis value. It increases until curvature variance exceeds .531 of circular (the median point of all possible minor axis values). For minor axii smaller than the median, Maclaurin “error” begins to decrease.

At “ε=.5” (minor axis .866 of major axis) curvature variance is 88.4% of circular. Maclaurin’s estimate is .519% too high. At “ε=.86603,” (minor axis .5 of major *or the median*) curvature variance is 53.1% of circular. Maclaurin’s estimate is 1.722% too high. Maclaurin’s greatest error is at the minor axis median.

Why is this so? Elliptical curvature variance is a function of the minor axis. A review of the quantum formula will reveal that the minor-axis radius modifies the curvature constant for all elliptical secants. Maclaurin’s “π only” constant (soon to be revealed) does not account for this actual change in curvature variance.

This is confirmed by Maclaurin error past the median minor axis. His error starts to decrease at “ε=.95.” Curvature variance is only 33.5% of circular (minor axis 31% of major axis). The Maclaurin “error” is only 1.61% greater than quantum (down from 1.722%). When “ε” reaches its maximum value of “1” and curvature variance is “0%” (minor axis equals “0”) Maclaurin’s “error” is only .5% and may actually be less if more differentials are used. Maclaurin error drops past the minor axis median point.

Determining Maclaurin’s Curvature Constant

Maclaurin’s Derivative Series for the estimation of elliptical circumference can be broken into two parts; the natural derivative and Maclaurin’s modification of the derivative factorials. It is the Maclaurin modification of the factorials, which determines his curvature constant.

$$\chi_e = 2\pi r_2\left(1-\sum_1^n \frac{1}{2n-1}\left(\frac{(2n)!}{\left(2^n n!\right)^2}\right)^2 \varepsilon^{2n}\right)$$

$$\text{natural derivative} = \frac{(2n)!}{n}\varepsilon^{2n}$$

$$\text{Maclaurin modification factorial} = \frac{n(2n)!}{2n-1\left(2^n n!\right)^4}$$

If the natural derivative is multiplied by the Maclaurin modification, the factorial for the elliptical series results. This resultant factorial determines the “curvature constant” operator for the Maclaurin Derivative Series. This is seen by applying the Maclaurin formula to an elliptical eccentricity of “1.”

When “$\varepsilon = 1$” all exponents “ε” also equal “1.” Since the derivative series is composed of exponents of eccentricity, eccentricity drops out as a factor at “$\varepsilon = 1$.” Only the factorials are remaindered. The value of the summation of derivative series becomes completely the value of the summation of the factorials alone.

Further, at “ε=1” we know what the value of the circumference must be. The ellipse has been compressed completely to the major axis and the value of the circumference must be four times the major radius ($4r_2$). The Maclaurin formula must be the following:

$$4r_2 = 2\pi r_2\left(1-\frac{\pi-2}{\pi}\right)$$

and therefore

$$\sum_1^n \frac{1}{2n-1}\left(\frac{(2n)!}{\left(2^n n!\right)^2}\right)^2 = \frac{\pi-2}{\pi}$$

Eccentricity has dropped out and the factorial sums to reveal Maclaurin's "curvature constant." This curvature constant is a "π only" version.

Unlike the quantum curvature constant which has quantum π in the denominator ($\pi_Q = 3$) the Maclaurin version has π itself in the denominator. Instead of applying curvature over linear distance, the Maclaurin constant applies curvature over curved distance.

The derivative series for all other eccentricity values are modifications of this "curvature constant." They are partials or fractions contributing to the summation of Maclaurin's curvature constant .

In summery:

The factor "$\frac{\pi-2}{\pi}$" is Maclaurin's curvature constant .

It is dissimilar to the quantum "$\frac{\pi-3}{3}$" constant in that it applies "π" to the denominator.

Maclaurin's curvature constant equals the summation of the factorials alone:

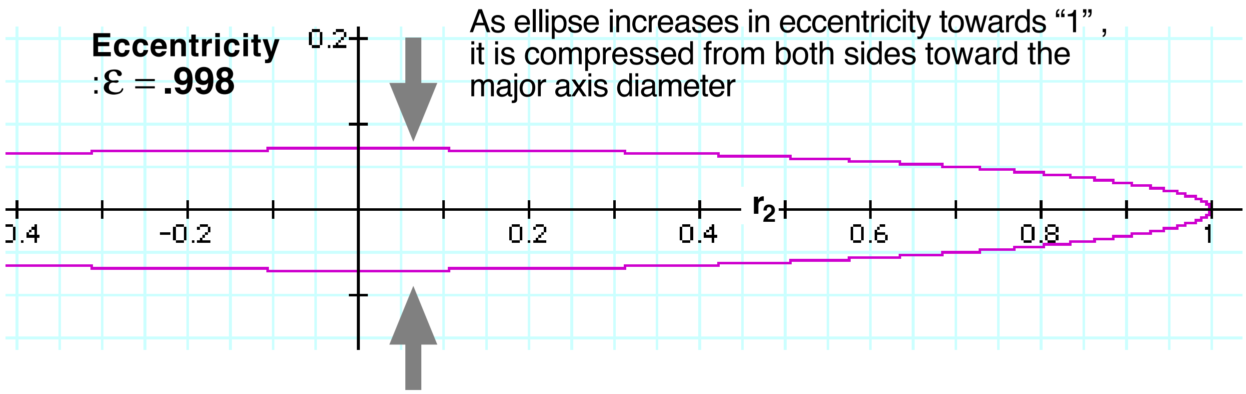
$$\frac{\pi-2}{\pi} = \sum_1^n \frac{1}{2n-1}\left(\frac{(2n)!}{\left(2^n n!\right)^2}\right)^2 = 0.3633802276$$

How Maclaurin's Curvature Constant Operates Mathematically

The equation "$4r_2 = 2\pi r_2\left(1-\frac{\pi-2}{\pi}\right) = 2\pi r_2 - \left(2\pi r_2 - 4r_2\right)$" identifies mathematical operation.

The factor "($2\pi r_2$ - $4r_2$)" is the variance between curved circumference and "diameter circumference.". "Diameter circumference" is the diameter as measured two dimensionally, form both above and below the diameter line.

As eccentricity approaches "1," the elliptical circumference approaches "$4r_2$" or twice the major axis diameter. This is diameter circumference. The value "$4r_2$" is obtained by compressing both curvatures from above and below the major axis to the axis. The linear "diameter circumference" is thus twice the diameter.



At “eccentricity=1,” elliptical circumference equals the diameter circumference of “$4r_2$.” Therefore, the statistic “$2\pi r_2 - 4r_2$” is the variance between circular curved circumference and this elliptical diameter circumference. It is the amount the curved circumference exceeds the linear circumference.

$2\pi r_2 - 4r_2$ = curved circumference of circle *minus* “diameter circumference.” When this is divided by circular circumference it gives the excess of curvature length to diameter length *per* unit of curvature:

$$\frac{2\pi r_2 - 4r_2}{2\pi r_2} = 1 - \frac{4r_2}{2\pi r_2} = 1 - \frac{2}{\pi} = \frac{\pi - 2}{\pi}$$; curvature excess *per* unit of curvature

Curvature excess *per* unit of curvature is Maclaurin statistical curvature constant to which his factorials sum:

$$\frac{\pi - 2}{\pi} = \sum_{1}^{n} \frac{1}{2n-1}\left(\frac{(2n)!}{\left(2^n n!\right)^2}\right)^2$$

Maclaurin’s “Curvature excess relative to the diameter per unit of curvature” is the *Euclidean curvature constant*. and the only curvature constant available to three dimensional geometry.

When multiplied by the circumference of a circle it produces the variance between curved circumference and non-curved “diameter circumference”:

$$2\pi r_2\left(\frac{\pi - 2}{\pi}\right) = 2\pi r_2 - 4r_2 \; ;$$

This is the excess of curvature over the only “linear circumference” (diameter circumference) which is available to strict Euclidean geometry. The only “linear circumference” available to strict Euclidean geometry is two dimensional curved circumference which has been compressed to a single dimension along the major axis. This, of course, is not true of Quantum construction which provides two-dimensional linear circumference.

For strict Euclidean geometry, linear circumference exists at one and only one elliptical value; at “ε=1” where two dimensional circumference has been suppressed to a single dimension. Therefore, Euclidean linear circumference requires the complete *negation* of the Maclaurin curvature constant. Since Maclaurin’s constant is “curvature *per* unit of curvature” and curvature requires two dimensions, Euclidean linear circumference requires that the Maclaurin constant equal “0.” The constant must be completely negated before Euclidean linear circumference can exist.

The multiplication of the negation of Maclaurin's constant by circumference does indeed produces the Euclidean linear circumference.:

$$2\pi r_2\left(1-\frac{\pi-2}{\pi}\right)=2\pi r_2\left(\frac{\pi-(\pi-2)}{\pi}\right)=4r_2$$

Maclaurin is arguing that by modifying his constant by the summation of derivatives he is differentiating the variance between curved and linear circumference to "0" and estimating the integration of that differential.

We can use linear equations to review how Maclaurin's summed factorials might work. If we modified the Maclaurin constant by a partial "1/ x" we get the following equation:
:

$$2\pi r_2\left(1-\left(\frac{1}{x}\right)\frac{\pi-2}{\pi}\right)=2\pi r_2\left(\frac{x\pi-(\pi-2)}{x\pi}\right)=\frac{2r_2\pi(x-1)+4r_2}{x}=2r_2\pi\frac{x-1}{x}+4r_2\frac{1}{x}$$

The equation exchanges linear components of circumference (a fraction of $4r_2$) with curved components of circumference (a fraction of $2\pi r_2$). As "x" gets larger and larger the linear component "$4r_2$" approaches "0" and the curved component "$2\pi r_2$" approaches unity(unity=1($2\pi r_2$).

In contrast, as "x" approaches "1" the linear component of circumference approaches "$4r_2$ " and the curved component ($2\pi r_2$) approaches "0." As "x" decreases the fraction gets larger. This results in a greater contribution to the linear component of circumference. It becomes more linear or "flatter."

This is the essential logic of the Maclaurin formula. It is an exchange of a partial of the linear circumference defined by the major axis with the partial of the curved circumference defined by the originating circle.

Maclaurin's Estimation of the Eccentricity Integral

The actual Maclaurin estimation of elliptical circumference cannot be reproduced by such a linear equation. The variance constant is actually being modified during the process of derivative summation.

Any one factorial "n" is modified by its equivalent exponent of eccentricity "ε^{2n}." The modified factorial is only a part of the whole summation. The modification of the whole is the summation of individual partial factorials as modified by equivalent eccentricity derivatives.

The Maclaurin equation is an estimation of an integral. It is an estimation of the what we have termed the "natural derivative of the fractional partial" times the "Maclaurin modification factorial."

Maclaurin's modification factorial is the derivative of an *unknown function* of eccentricity, but an *unknown function* which incorporates Maclaurin's variance constant.

The circumference of the ellipse is thus the integration of the modification factorial *times* change in eccentricity from "eccentricity=0" to "eccentricity=eccentricity."

This proposed integral is nonfunctioning. For eccentricity to equal "0" the value of "x" in the denominator must equal “infinity,” a non-real number. The Maclaurin Derivative Series estimates this nonfunctioning integral.

Maclaurin, however, cannot get away from the fact that his proposed curvature variance constant,

$$“\left(\frac{\pi-2}{\pi}\right)”$$

is in the unknown function which supposedly produces circumference from elliptical eccentricity. Maclaurin’s unknown function must be the following:

$$\chi = 2\pi r_2\left(1 - f(\varepsilon_n)\,\frac{\pi-2}{\pi}\right)$$

But can Maclaurin’s estimation of the integral accurately do so? This is a fair question. Modern mathematics is so enthralled by the admitted brilliance of Maclaurin Derivative Series estimations of unknown integrals that they never question the accuracy of the integrals being estimated.

Maclaurin’s proposed curvature variance constant—the core of the unknown function —is not completely functional. It is the only curvature variance constant available to strict three-dimensional Euclidean geometry and therefore the only constant available to Maclaurin.

However, I have shown that the modified Maclaurin constant exchanges curved circumference, as defined by the circle, with linear circumference, as defined by the major axis. The major axis provides the only linear circumference available to non-quantum geometry. The *unknown function* for eccentricity, regardless of its composition, must use the major axis as the measure of linear circumference.

The Maclaurin Derivative Series estimation of the eccentricity integral for circumference is based upon the Euclidean curvature variance constant and must introduce error between “$\varepsilon = 0$” and “$\varepsilon = 1$” just as the data table above has shown. The Euclidean restriction forces elliptical curvature variance to be a function of the major axis, not the minor axis. The Maclaurin formula is built upon in incorrect curvature variance constant.

Quantum geometry has proven that elliptical curvature variance is a function of the minor axis, not the major axis. This conclusion is based upon straight forward geometric proofs from the analytic graph of the ellipse, as intersected by the quantum dimension (*see appendix*). It is not based upon intuitive speculation.

Elliptical curvature as a ratio of the circle’s curvature closely follows the ratio of the minor axis to the major axis.

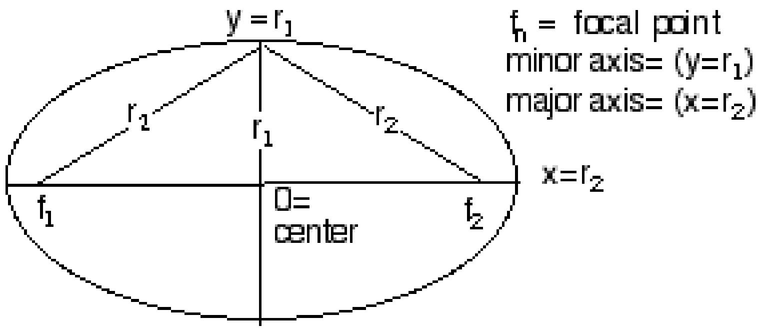
Eccent.	**Var/Circ**	**Mac. error**	**r_1 / r_2**
0.25	*0.973*	***+.120%***	0.968
0.5	*0.884*	***+.519%***	0.866
0.866	*0.531*	***+1.722***	0.5
0.95	*0.335*	***+1.61***	0.312

This correspondence between elliptical variance and the minor axis represents the quantum geometric discovery that elliptical curvature varies by minor axis length. The covariance is an artifact of the quantum formula itself. It is presented to demonstrate that the Maclaurin error

Appendix

Determining Circumference of an Ellipse by Quantum Construction

Technically, an ellipse is a quantum construction. Specifically, the minor axis of the ellipse is a quantum inexact partial of the major axis. The differentiator for this partial is the focal point. The actual equations are the following:



"f" is always a fractional proportion or percentage of "r_2"

Therefore, let $f = \frac{r_2}{x_n}$; x_n = any number greater than "1"

$$r_1^2 = r_2^2 - f^2 = r_2^2 - \left(\frac{r_2}{x_n}\right)^2 = \left(1 - \frac{1}{x_n^2}\right) r_2^2$$

$$\mathbf{r_1^2 = \left(\frac{x_n^2 - 1}{x_n^2}\right) r_2^2}$$

The model for this is the Inexact Differentiation of Measure available to a quantum:

$$\textbf{Inexact Differential of Q} = \left(\frac{n^2 - 1}{n^2}\right) Q^2 \ ; \ \textbf{Q = quantum}$$ [8]

The two equations above have a similarity of form. The difference between the Euclidean value "x_n" and the quantum value "n" is that the Euclidean values between two quantum points "n" and "n+1" are described by "x_n."

[8] The inexact quantum differential is built upon the premise that any unit of measure cannot be subdivided into equal units by simple whole number division. Division by most values of "n" produces inexact and irrational numbers which cannot produce an exact number (n) of equal subdivisions for the unit. However, subtracting the "Inexact" subdivision (1/ n) from the whole produces a new unit of measure which is an exact fraction of the original. The inverse of any proportion, "1/ n," is, "1-1/n." The inverse of the fraction produces an new unit of measure which is an exact fraction of the original.

However, dividing a unit of measure by a whole number "n," does not automatically produce the inverse of the proportions The derivative of the fractional function, however, does produce the first derivative of its inverse. The derivative of "1/n" is the following:

Differentials of Partials *

$$D^1(1/x) = 1 - 1/x^2$$

$$D^2(1/x) = 2/x^3$$

$$D^n(1/x) = \mp n!/x^{n+1}$$

In his equation for the circumference of an ellipse, Collin Maclaurin showed that the derivative for a proportional variable produces a function of the variable's inverse. While the equation itself contains systematic error, the principle of the derivative series for proportions alternate between the variable and its inverse is contained in that equation. Henceforth, alternating derivatives of proportionals will be designated "The Maclaurin Differential Series.**" The Maclaurin discovery was crucial to the development of quantum mathematics.*

Derivative series of a partial

$$\varepsilon = \text{eccentricity} = \frac{f}{r_2} = \text{ a partial of the major axis} = \sqrt{\frac{r_2^2 - r_1^2}{r_2^2}} \;\; ; \;\; \varepsilon^2 = \frac{r_2^2 - r_1^2}{r_2^2}$$

$$\frac{\chi_e}{2\pi r_2} = 1 - \sum_1^n \frac{1}{2n-1}\left(\frac{(2n)!}{(2^n n!)^2}\right)^2 \varepsilon^{2n} \quad ; n = \text{"nth" derivative of the partial "}\varepsilon\text{"}$$

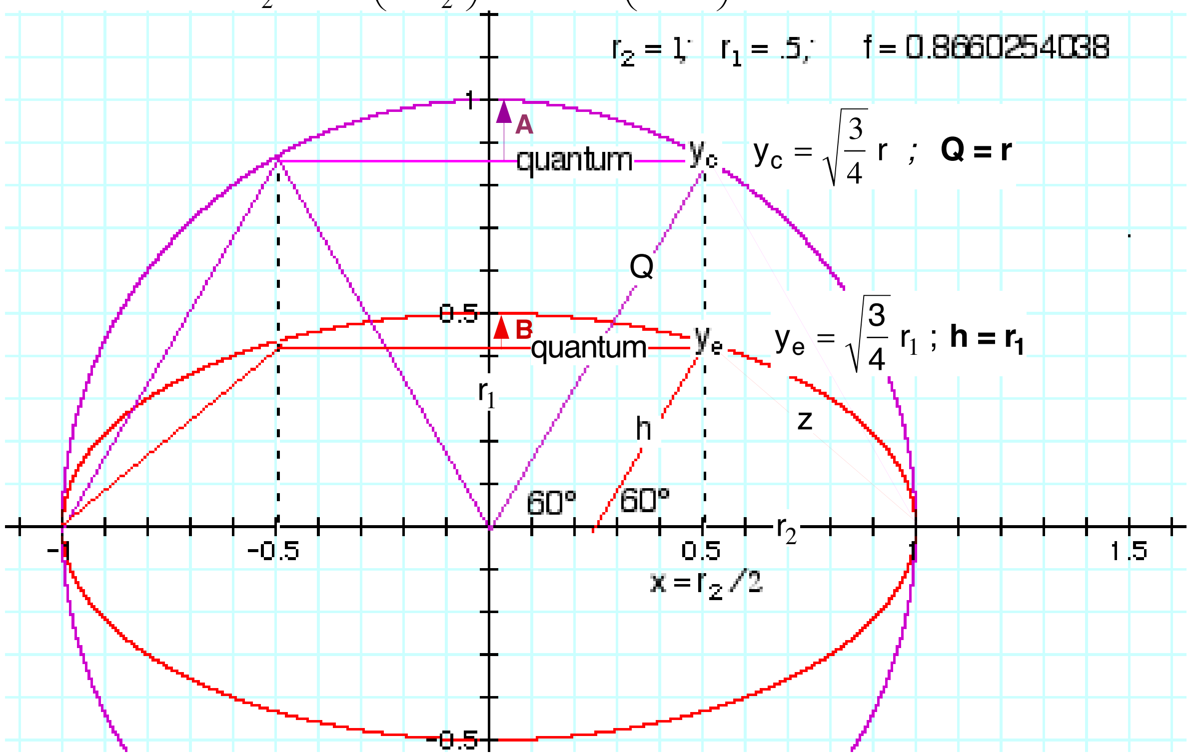
$$\chi_e = (2\pi r_2)\left[1 - \sum_1^n \frac{1}{2n-1}\left(\frac{(2n)!}{(2^n n!)^2}\right)^2 \varepsilon^{2n}\right]$$

Expansion of Maclaurin Factorial for "n = 2."

$$\frac{(1**3*)(1**3*)}{(**1)(**1)(2^2*2*1)^2}\frac{\varepsilon^4}{3} = \frac{1}{2n-1}\left(\frac{(2n)!}{(2^n n!)^2}\right)^2 \varepsilon^{2n}$$

circle: $y^2 = r^2 - x^2$; $y^2 = 1^2 - x^2$; $r = 1$

ellipse: $y^2 = \frac{r_1^2 r_2^2 - r_1^2 x^2}{r_2^2} = \left(1 - \frac{x^2}{r_2^2}\right) r_1^2$; $y^2 = \left(1 - \frac{x^2}{1}\right) .5^2$; $r_1 = .5$; $r_2 = 1$ $(r_2 = r)$



$$y_c = \sqrt{\left(1 - \frac{x^2}{r^2}\right) r^2} = \sqrt{\left(1 - \frac{r^2/2^2}{r^2}\right) r^2} = \sqrt{\frac{3}{4}}\, r = \sin 60°(r) \quad ; \quad \therefore \mathbf{Q = r}$$

$$y_e = \sqrt{\left(1 - \frac{x^2}{r_2^2}\right) r_1^2} = \sqrt{\left(1 - \frac{r_2{}^2 / 2^2}{r_2^2}\right) r_1^2} = \sqrt{\left(1 - \frac{1}{2^2}\right) r_1^2}$$

$$= \sqrt{\frac{3}{4}\, r_1^2} = \sqrt{\frac{3}{4}}\, r_1 = \sin 60°(r_1) \quad ; \quad \therefore \mathbf{h = r_1}$$

$$z^2 = \frac{3}{4}\, r_1^2 + \frac{r_2^2}{4} = \frac{3\, r_1^2 + r_2^2}{4}$$

quantum circumference of ellipse $= 4\sqrt{\frac{3\, r_1^2 + r_2^2}{4}} + \sqrt{4\, r_2^2} = 2\sqrt{3\, r_1^2 + r_2^2} + 2\, r_2$

.

Statistical Curvature Variance

The secant values of the illustrated ellipse are the following:

Four (4) secants at a value of "$\sqrt{\frac{3r_1^2 + r_2^2}{4}}$" and two (2) secants at a value of "r_2."

(See illustration above)

The curved lines circumscribing these secants and thus composing the actual circumference of the ellipse can be determined by a statistic I will call “curvature variance.” .

Statistical curvature variance is determined by the quantum defined circle. In the above illustration, the circle is intersected by the quantum dimension from outside the plane which contains the circle. This intersecting quantum dimension imposes a quantum definition upon the circle which is represented by purple lines. The quantum secant which intersects the vertical “y” axis imposes a distance value along the x axis which is equal to the radius and which is located on “x” by “ $x = \pm r_2 / 2$.” This is true for all ellipses, not just the one illustrated. Quantum elliptical parameters are completely mathematically regular.

The curvature variance between the circle perimeter and the secant is defined by the y axis radius. The value “(π-3)/ 3” is the statistic which defines the curvature variance per secant:

$$\text{statistical curvature variance} = \left(\frac{\pi - 3}{3}\right)$$

A quantum circle is composed of (6) 60° angles. The secant of each angle is equal to the radius. Quantum circumference is defined as the summation of these secants. The quantum circumference is thus “6r.” Pi is defined as the ratio of circumference to the diameter of the circle. Therefore quantum pi (π_Q) is equal to “3.”

The curved circumference is, of course, 2π r. The curvature variance for each secant, therefore, is :

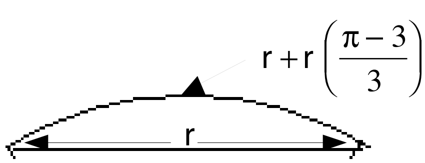
$$(\text{statistical curvature variance}) \times (\text{secant}) + \text{secant} = \frac{2\pi r}{6} = \frac{\pi r}{3} \quad ; \text{secant} = \text{radius}$$

$$(\text{statistical curvature variance}) \times (\text{secant}) = \frac{\pi r}{3} - r = r\left(\frac{\pi - 3}{3}\right)$$

$$\text{secant} = r$$

$$\therefore (\text{statistical curvature variance}) = \left(\frac{\pi - 3}{3}\right)$$

$$r + r\left(\frac{\pi - 3}{3}\right) = \frac{\pi r}{3} = \text{ length of curved line for one secant.}$$

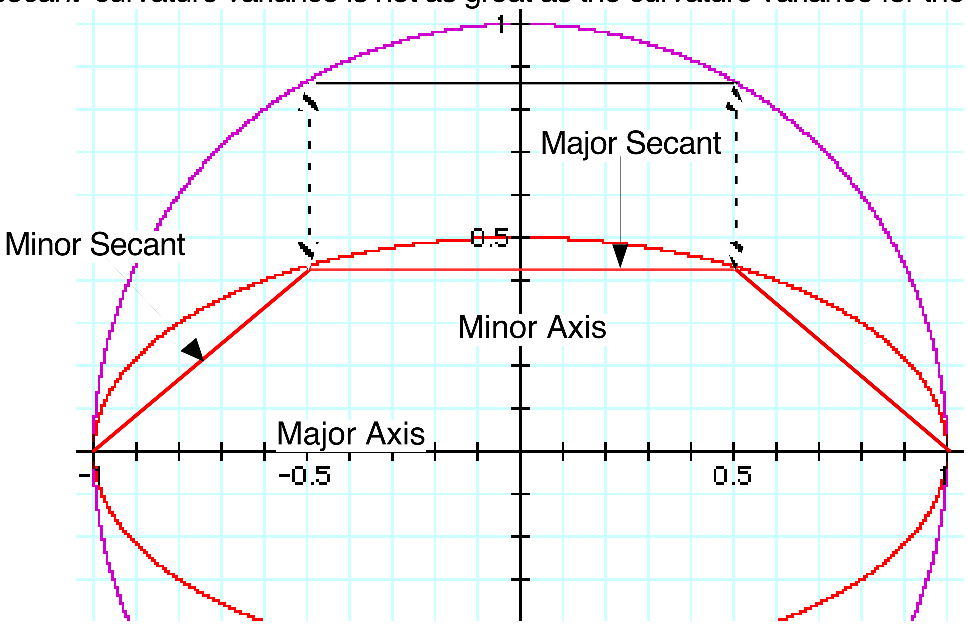


Curvature Variance for the Ellipse

Curvature variance as defined above is a quantum statistic. It is a quantum statistic which represents the height of the curved line relative to the quantum secant.

The quantum secant in the ellipse which is parallel to the major axis is the *major secant*. Just as the ellipse does not change the radial length of the major axis, so the circular length of the *major secant* is not changed by the ellipse. However, the statistical curvature variance does change.

With the ellipse, the variance of the curved line relative to the *major secant* changes. In the above illustration, height "B" for the ellipse is not as great as height "A" for the circle. The *major secant* curvature variance is not as great as the curvature variance for the circle secant.



The curvature variance for the major secant is only a proportion of the curvature variance for the circle. We can determine what this proportion is by the following equation:

$$\frac{B}{A} = \frac{r_1 - \sqrt{3/4}\, r_1}{r_2 - \sqrt{3/4}\, r_2} = \frac{r_1\left(1 - \sqrt{3/4}\right)}{r_2\left(1 - \sqrt{3/4}\right)} = \frac{r_1}{r_2}$$

$$\text{statistical curvature variance of } \textit{major secant} = \frac{r_1}{r_2}\left(\frac{\pi - 3}{3}\right)$$

That is, the statistical curvature variance has been reduced in proportion to the reduction in height between the secant and the curved line for the ellipse. The length of the curved line for one of the two elliptical *major secants* is the following:

$$(\text{statistical curvature variance}) \times (\text{secant}) = \frac{r_1}{r_2}\left(\frac{\pi - 3}{3}\right) r_2 = r_1\left(\frac{\pi - 3}{3}\right)$$

$$\text{length of curved line for } \textit{major secant} = r_1\left(\frac{\pi - 3}{3}\right) + r_2$$

There are two *major secants* in the ellipse. Therefore the portion of the elliptical circumference which subscribes the *major secants* is the following:

***Major Secant* Circumference**

$$\chi_{major} = 2\left(r_1\left(\frac{\pi-3}{3}\right)+r_2\right)$$

The Quantum *Minor Secants*

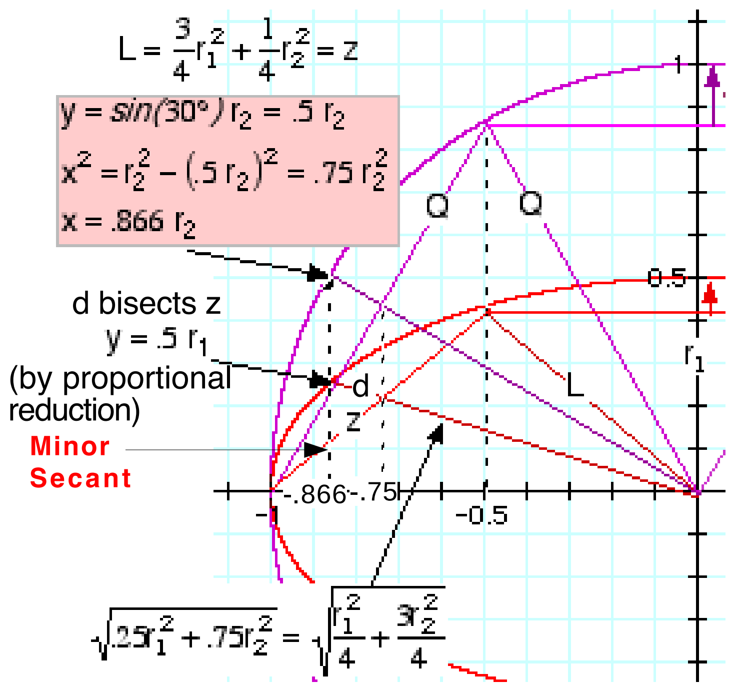
The quantum *minor secants* are the four secants which terminate on the major axis, the lengths of which vary with elliptical eccentricity. The lengths of the *minor secants* are mathematically regular with respect to any ellipse. That length is given by the following:

$$(\textit{minor secant})^2 = \frac{3}{4}\ r_1^2 + \frac{r_2^2}{4} = \frac{3\ r_1^2 + r_2^2}{4}$$

See illustration above.

$$\textit{minor secant} = \frac{\sqrt{3\ r_1^2 + r_2^2}}{2}$$

The Quantum Construction of the *Minor Secant*



The *minor secants* intersect the y axii at "$x = \pm\ r_2 / 2$" and the x axis at "y=0." The circular *major secant* is compressed along the y axis to form the elliptical *major secant*. While the length of the *major secant* is not changed, this is not true of the *minor secant.* The value of the *minor secant* is compressed from "$Q = r_2$ (above)" to:

$$z = \frac{\sqrt{3\ r_1^2 + r_2^2}}{2} \quad \text{(also above)}$$

The length of the *major secant* is determined solely by the major axis. However, the length of the *minor secant* is determined by both the major and minor axis with greater weight given the minor axis. The minor axis factor "$3\ r_1^2$" under the square-root symbol contributes more than the major axis factor "r_2^2" does. The *minor secant* length is a function of both the minor and major axii with greater weight given the minor axis.

The reverse is true of the elliptical radius which determines curvature variance for the *minor secant*. The elliptical radius marked "d" in the above illustration determines curvature variance. It is the resultant bisecting radius when the circular *minor secant* is compressed down to become the elliptical *minor secant*. The radius which bisects the secant is used to determine curvature variance.

The length of the radius bisecting the elliptical *minor secant* is the following:

$$d = \frac{\sqrt{r_1^2 + 3\ r_2^2}}{2} \quad \text{(see above)}$$

The bisecting radius changes much more slowly than does the elliptical *minor secant* because more weight is given the major axis than the minor axis in the formula. The minor axis factor "r_1^2" under the square-root symbol contributes less than the major axis factor "$3 r_2^2$" does. Since the major axis is constant, the length of the bisecting radius changes more slowly.

Determining Curvature Variance for the *Minor Secant*

Since the curvature variance for any ellipse is changed by the compression of a circle along the minor y axis alone, the minor axis determines that curvature variance.

I have shown that curvature variance for the *major secant* is determined by the proportionality which the minor axis, "r_1" makes of the major axis "r_2." This is possible because the bisecting radius which determines curvature variance have the same "x" value for both the ellipse and its originating circle. The minor axis lies on the bisecting radius which has a single "x=0" value and therefore can change along the "y" axis without any change in x.

For the curvature variance of the *minor secant,* however, this is not true. The minor axis cannot be a proportional component of the major "x" axis alone because the bisecting radial does not have freedom with respect to the x axis. Changes in the radial require changes in "x." To determine curvature variance for the *minor secant*, therefore, the minor axis must be made a proportion of the bisecting radius itself . Statistical curvature variance is the following:

$$\text{Statistical curvature variance of } \textit{minor secant} = \frac{r_1}{d} = \frac{r_1}{\dfrac{\sqrt{r_1^2 + 3\ r_2^2}}{2}} \left(\frac{\pi - 3}{3}\right)$$

$$= \frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right)$$

Only when the minor axis "r_1" equals the major axis "r_2" will the minor axis be proportional to the major axis alone and that proportion will always be "1" (i.e. the ellipse is a circle). Therefore, the statistical curvature variance for the *minor secant* must be modified by the above proportion.

The statistical curvature variance *per secant* is the modified curvature variance statistic times the secant. Now the secant which the statistic is determining is the minor secant. . The formula is the following:

$$(\text{Statistical curvature variance}) \times (\text{secant}) = \frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right) \frac{\sqrt{3\ r_1^2 + r_2^2}}{2}$$

The curved line subscribing any one minor secant is the following:

χ_{min} = curved line subscribing the *minor secant*

χ_{min} = (cuvature variance *per* secant) + (secant length)

$$\chi_{min} = \frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right) \frac{\sqrt{3\ r_1^2 + r_2^2}}{2} + \frac{\sqrt{3\ r_1^2 + r_2^2}}{2} = \frac{\sqrt{3\ r_1^2 + r_2^2}}{2} \left(\frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right) + 1\right)$$

Determining the Exact Formula for the Periphery of the Ellipse

Because quantum geometry has provided exact information upon the curvature variances for elliptical secants, we can produce an exact formula for the periphery or circumference of any ellipse.

Every ellipse has four *minor secants* with a total curvatured line or periphery value as follows:

$$\chi_{minor} = 2\sqrt{3\ r_1^2 + r_2^2} \left(\frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right) + 1\right)$$

Every ellipse has two *major secants* with a total periphery value of:

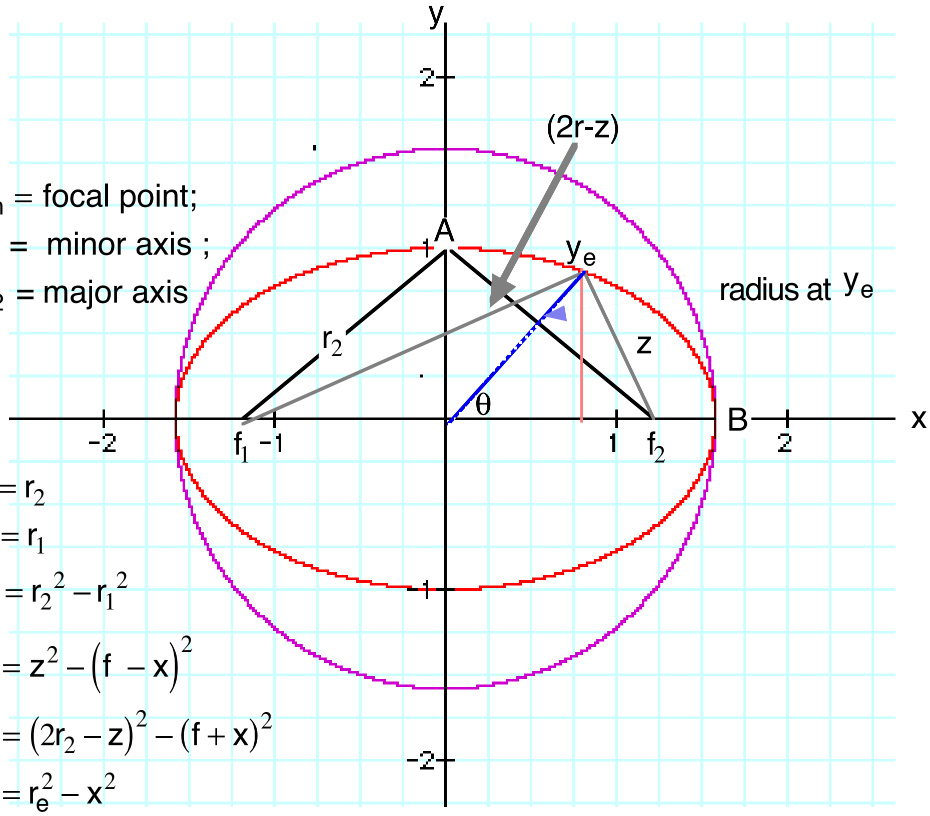
$$\chi_{\text{major}} = 2\left(r_1 \left(\frac{\pi - 3}{3}\right) + r_2\right)$$

Total Periphery of Ellipse

$$\chi_{\text{total}} = 2\sqrt{3\ r_1^2 + r_2^2} \left(\frac{2\ r_1}{\sqrt{r_1^2 + 3\ r_2^2}} \left(\frac{\pi - 3}{3}\right) + 1\right) + 2\left(r_1 \left(\frac{\pi - 3}{3}\right) + r_2\right)$$

$$\text{circle} = 4\ r + 4r\left(\frac{\pi - 3}{3}\right) + 2r + 2r\,\frac{\pi - 3}{3} = 6r + 6r\left(\frac{\pi - 3}{3}\right) = 6\ r + 2r(\pi - 3) = 2\pi r$$

The Conventional Elliptical Construction



f_n = focal point;
r_1 = minor axis ;
r_2 = major axis

$$B = r_2$$
$$A = r_1$$
$$f_n^2 = {r_2}^2 - {r_1}^2$$
$$y_e^2 = z^2 - (f - x)^2$$
$$y_e^2 = (2r_2 - z)^2 - (f + x)^2$$
$$y_e^2 = r_e^2 - x^2$$
$$y_e^2 = r_e^2 \, sin^2(\theta)$$
$$\mathbf{x^2 = r_e^2 - y_e^2 = r_e^2\left(1 - sin^2(\theta)\right)}$$
$$z^2 - f^2 + 2fx - x^2 = 4{r_2}^2 - 4r_2 z + z^2 - f^2 - 2fx - x^2$$
$$4fx = 4{r_2}^2 - 4r_2 z$$
$$4r_2 z = 4{r_2}^2 - 4fx$$
$$z = \frac{{r_2}^2 - fx}{r_2}$$
$$r_e^2 - x^2 = \left(\frac{{r_2}^2 - fx}{r_2}\right)^2 - (f - x)^2$$
$$r_e^2 - x^2 = \frac{{r_2}^4 - 2{r_2}^2 fx + f^2x^2}{{r_2}^2} - f^2 + 2fx - x^2$$
$$r_e^2 = \frac{{r_2}^4 + f^2x^2 - f^2{r_2}^2}{{r_2}^2} = \frac{{r_2}^4 + ({r_2}^2 - {r_1}^2)x^2 - ({r_2}^2 - {r_1}^2){r_2}^2}{{r_2}^2} = \frac{{r_2}^2x^2 - r_1^2x^2 + r_1^2r_2^2}{{r_2}^2} = x^2 + r_1^2 - \frac{r_1^2x^2}{{r_2}^2}$$
$$\mathbf{y_e^2 = r_e^2 - x^2 = x^2 + r_1^2 - \frac{r_1^2x^2}{{r_2}^2} - x^2 = \frac{r_1^2{r_2}^2 - r_1^2x^2}{{r_2}^2}}$$

Conventional Formula

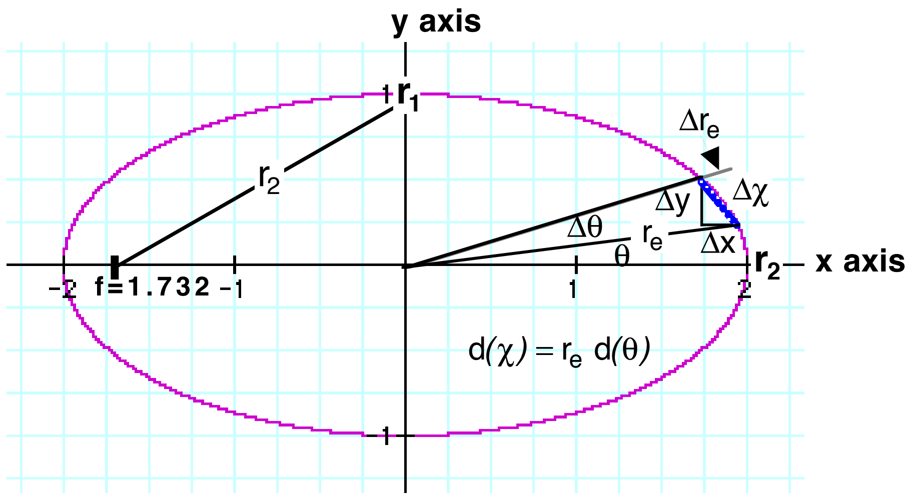
$$\frac{x^2}{r_2^2} + \frac{y^2}{r_1^2} = 1 \;;\; \frac{y^2}{r_1^2} = 1 - \frac{x^2}{r_2^2} \;;\; \mathbf{y^2 = \frac{r_1^2r_2^2 - r_1^2x^2}{r_2^2}}$$

The Ellipse as Graphed by a Single Radius

f = focal point ; r_1 = minor axis ; r_2 = major axis

$r_1 = 1$; $r_2 = 2$; $f = \sqrt{r_2^2 - r_1^2} = \sqrt{3} = 1.732$

$$y^2 = \frac{r_1^2 r_2^2 - r_1^2 x^2}{r_2^2} = \frac{4 - x^2}{4} = 1 - .25x^2$$



$$d(x) = -r_e \, sin(\theta) \, d(\theta) \;\; ; \; d(y) = r_e \, cos(\theta) \, d(\theta)$$

$$d(\chi)^2 = d(x)^2 + d(y)^2 = r_e{}^2 \, sin(\theta)^2 \, d(\theta)^2 + r_e{}^2 \, cos(\theta)^2 \, d(\theta)^2$$

$$d(\chi)^2 = r_e{}^2 \left(sin(\theta)^2 + cos(\theta)^2\right) d(\theta)^2 = r_e{}^2 (1) \, d(\theta)^2 \; ; \; \left\{sin(\theta)^2 + cos(\theta)^2 = 1\right\}$$

$$d(\chi) = \sqrt{r_e{}^2 (1) \, d(\theta)^2} = r_e \, d(\theta) d(\chi) = \sqrt{r_e{}^2 (1) \, d(\theta)^2} = r_e \, d(\theta)$$

$$\chi = \int_{\theta=0}^{360°} r_e \, d(\theta)$$

For the ellipse, the change in circumference is a function of change in elliptical radius as well as a function of change in angle. The radius, "r_e" decreases in length as the angle "θ" increases. Change in circumference "$\Delta\chi$" decreases for every change in angle "$\Delta\theta$." As "r_e" rotates between "0°" and "90°" change in circumference "$\Delta\chi$" becomes smaller and smaller.

This can be mathematically demonstrated. An equation for "r_e" as a function of sine/cosine of "θ" is derived. It is the following: $r_e = \sqrt{\dfrac{r_1^2 \, r_2^2}{r_1^2 \, cos^2(\theta) + r_2^2 \, sin^2(\theta)}}$

(for proof of equality see below)

As the angle "θ" moves from "0°" to "90°" , cosine ranges from "1" to "0." Sine ranges from "0" to "1." At "θ=0° (i.e. radius laying along the x axis), $cos(\theta) = 1$ and $sin(\theta) = 0$.

The denominator in the above equation becomes "r_1^2" and $r_e = r_2$.

At 90°(i.e. radius laying along the y axis), the denominator becomes "r_2^2" and $r_e = r_1$.

As the angle changes between "0°" and "90°." the elliptical radius "r_e" decreases from a maximum of : $r_e = r_2$ with lower limit of: $r_e = r_1$

Single Elliptical Radius as Function of Sine/Cosine

$$\frac{r_e^{\,2}\left(1-\sin^2(\theta)\right)}{r_2^2}+\frac{r_e^{\,2}\sin^2(\theta)}{r_1^2}=1$$

$$r_e^{\,2}\left(\frac{1-\sin^2(\theta)}{r_2^2}+\frac{\sin^2(\theta)}{r_1^2}\right)=1$$

$$r_e^{\,2}=\frac{1}{\left(\frac{1-\sin^2(\theta)}{r_2^2}+\frac{\sin^2(\theta)}{r_1^2}\right)}=\frac{1}{\frac{r_1^2\left(1-\sin^2(\theta)\right)+r_2^2\sin^2(\theta)}{r_1^2r_2^2}}$$

$$r_e^{\,2}=\frac{r_1^2r_2^2}{r_1^2\left(1-\sin^2(\theta)\right)+r_2^2\sin^2(\theta)}$$

$$\mathbf{r_e=\sqrt{\frac{r_1^2r_2^2}{r_1^2\left(1-\sin^2(q)\right)+r_2^2\sin^2(q)}}}$$

also

$$y=r_e\sin(\theta);\quad y^2=r_e^2\sin^2(\theta)$$

$$y^2=\frac{r_1^2\,r_2^2-r_1^2r_e^2\left(1-\sin^2(\theta)\right)}{r_2^2}$$

$$r_e^2=\frac{r_1^2\,r_2^2-r_1^2r_e^2\left(1-\sin^2(\theta)\right)}{r_2^2\sin^2(\theta)}$$

$$r_e^2r_2^2\sin^2(\theta)=r_1^2\,r_2^2-r_1^2r_e^2\left(1-\sin^2(\theta)\right)$$

$$r_e^2r_2^2\sin^2(\theta)+r_1^2r_e^2\left(1-\sin^2(\theta)\right)=r_1^2\,r_2^2$$

$$r_e^2=\frac{r_1^2r_2^2}{r_1^2\left(1-\sin^2(\theta)\right)+r_2^2\sin^2(\theta)}$$

NOTE: equation confirmed

$$\frac{1}{r_e^2}=\frac{r_1^2\left(1-\sin^2(\theta)\right)+r_2^2\sin^2(\theta)}{r_1^2r_2^2}=\frac{\left(1-\sin^2(\theta)\right)}{r_2^2}+\frac{\sin^2(\theta)}{r_1^2}=\frac{\left(\cos^2(\theta)\right)}{r_2^2}+\frac{\sin^2(\theta)}{r_1^2}$$

Additional Equations for the Ellipse

The Full Calculus Equation for the Periphery of an Ellipse

$$2\sqrt{3\,r_1^2+r_2^2}\left(\frac{2\,r_1}{\sqrt{r_1^2+3\,r_2^2}}\left(\frac{\pi-3}{3}\right)+1\right)+2\left(r_1\left(\frac{\pi-3}{3}\right)+r_2\right)=\int_{\theta=0}^{360^\circ}\sqrt{\frac{r_1^2\,r_2^2}{r_1^2\,cos^2(\theta)+r_2^2\,sin^2(\theta)}}\,d(\theta)$$

Curvature variance per ellipse

$$\text{formula}=\frac{\text{Total curvature variance}}{\text{Total secant circumference}}$$

$$\text{minor curvature variance}=\left(\frac{2\,r_1\left(2\sqrt{3\,r_1^2+r_2^2}\right)}{\sqrt{r_1^2+3\,r_2^2}}\left(\frac{\pi-3}{3}\right)\right)$$

$$\text{Major curvature variance}=2\,r_1\left(\frac{\pi-3}{3}\right)$$

$$\text{Total Curvature variance}=\left(\frac{2\,r_1\left(2\sqrt{3\,r_1^2+r_2^2}\right)}{\sqrt{r_1^2+3\,r_2^2}}\left(\frac{\pi-3}{3}\right)\right)+2\,r_1\left(\frac{\pi-3}{3}\right)$$

$$=2\,r_1\left(\frac{\pi-3}{3}\right)\left(\frac{\left(2\sqrt{3\,r_1^2+r_2^2}\right)}{\sqrt{r_1^2+3\,r_2^2}}+1\right)$$

$$\text{Total secant circumference}=2\sqrt{3\,r_1^2+r_2^2}+2\,r_2$$

$$\frac{\text{Total curvature variance}}{\text{Total secant circumference}}=\frac{2\,r_1\left(\frac{\pi-3}{3}\right)\left(\frac{\left(2\sqrt{3\,r_1^2+r_2^2}\right)}{\sqrt{r_1^2+3\,r_2^2}}+1\right)}{2\sqrt{3\,r_1^2+r_2^2}+2\,r_2}$$

Maclaurin Data: **"Differentiations Needed for Significance"**

$\varepsilon = 1$	$\varepsilon = .866$	$\varepsilon = .5$	$\varepsilon = .25$
0.25	0.1875	0.0625	0.015625
0.046875	0.0263671875	0.0029296875	0.00018310547
0.01953125	0.0082397461	0.00030517578	0.00000476837
0.0106811523	0.0033795833	0.00004172325	0.00000016298
0.006729126	0.0015968531	0.00000657141	0.00000000642
0.0046262741	0.0008233774	0.00000112946	0.00000000028
0.0033752918	0.00045054707	0.00000020601	**Total=**0.0158130435:**N=6**
0.0025710231	0.00025739262	0.00000003923	
0.0020234904	0.00015193314	0.00000000772	
0.0016339685	0.00009201451	0.00000000156	
0.0013470112	0.0000568912	0.00000000032	
0.001129525	0.00003577923	0.00000000007	
0.00096076463	0.00002282514	0.00000000001	
0.00082718893	0.00001473881	**Total=**0.06578454233:**N=13**	
0.00071965437	0.00000961707		
0.00063180594	0.00000633234		
0.00055911546	0.00000420284		
0.00049828577	0.00000280919		
0.00044686986	0.00000188949		
0.00040302075	0.00000127806		
0.00036532323	0.00000086889		
0.00033267813	0.00000059343		
0.00030422126	0.000000407		
0.00027926561	0.00000028021		
0.00025725948	0.0000001936		
0.00023775571	**Total=**0.22901734125**: N=25**		
0.00022038878			
0.00020485755			
0.00019091214			
0.00017834376			
0.00016697689			
0.00015666313			
0.00014727629			
0.00013870853			
0.00013086726			
0.00012367259			
0.00011705533			
0.00011095532			
0.00010532002			
0.00010010339			
0.00009526496			
0.00009076904			
0.00008658404			
0.00008268194			
0.00007903781			
0.0000756294			
0.00007243681			
0.00006944218			
0.0000666295			
0.00006398431			
Total=0.36022088249**: N=50**			